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Planck-Curvature Thresholds and the Limits of Geometric Description

in Regular Black-Hole Interiors

Technical Report

based on the paper: "Planck-Curvature Thresholds and the Limits of Geometric Description in Regular Black Hole Interiors"

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Executive Summary

This report presents the results of an analysis of a known exponential regular black-hole metric in which the transition scale to the regular interior is not introduced arbitrarily, but is determined directly from the invariant curvature of the exterior Schwarzschild geometry.

The principal result is the demonstration that an appropriate relation between the transition radius and the mass of the object produces a global bound on the Kretschmann scalar that is independent of the asymptotic mass. Increasing the mass therefore enlarges the transition region without increasing the maximum curvature of the regular core.

PRINCIPAL RESULT
The exterior Schwarzschild geometry contains sufficient invariant information to determine the transition scale, and the same scaling relation produces the global curvature bound $K(r) \le K(0) = 2\eta K_p$.

TRANSITION SCALE $r_c \propto M^{1/3}$	GLOBAL BOUND $K(r) \le 2\eta K_p$	REGIMES 2 horizons / 1 / 0
INNER HORIZON $\kappa_- > 0$ away from extremality	LARGE-MASS LIMIT $r_- \sim \ell_p$	ENERGY CONDITIONS NEC and WEC satisfied

1. Scientific Context

The classical Schwarzschild solution describes the exterior geometry of a static, spherically symmetric mass. Its formal continuation toward the centre, however, produces divergent curvature invariants. In this work, that divergence is treated as a signal of the limit of applicability of the unmodified classical description rather than as a complete physical description of the deepest interior.

$$K_{Sch}(r) = 12\, r_s^2 / r^6 \quad (1)$$

Regular black-hole models replace the singularity with a finite-curvature core. A standard regularity condition is that the mass function satisfy $m(r) \sim r^3$ near the centre. A persistent unresolved issue is how to determine the scale at which the classical geometry should be replaced by an effective description.

2. Determining the Transition Scale from the Exterior Geometry

The transition radius r_c is defined as the point at which the Kretschmann scalar of the unmodified Schwarzschild geometry reaches a chosen curvature threshold. The convention $K_p = \ell_p^{-4}$ is adopted as the reference Planck-curvature scale, while the positive parameter η specifies its chosen normalization.

$$K_{Sch}(r_c) = \eta K_p \quad (2)$$

$$r_c^3 = \sqrt{(12/\eta)} \cdot r_s \ell_p^2 \quad (3)$$

Because the Schwarzschild radius is proportional to mass, the relation implies the scaling $r_c \propto M^{1/3}$. For macroscopic objects, this yields the hierarchy $\ell_p \ll r_c \ll r_s$. The Planck-curvature threshold may therefore be reached at a radius much larger than the Planck length itself, while still lying deep inside the outer horizon.

$$x(r) = r^3/r_c^3 = \sqrt{[\eta K_p / K_{Sch}(r)]} \quad (4)$$

The variable x has a direct geometric interpretation: $x \gg 1$ corresponds to the low-curvature region, $x = 1$ marks the centre of the transition region, and $x \ll 1$ identifies the domain in which an unmodified Schwarzschild continuation would require curvature above the adopted threshold.

3. Exponential Regular Metric

The analysis employs a known exponential mass profile. The novelty lies not in the metric itself, but in determining its transition scale from the exterior geometry.

$$m(r) = M [1 - \exp(-r^3/r_c^3)] = M P(x) \quad (5)$$

$$P(x) = 1 - e^{-x} \quad (6)$$

Near the centre, the profile behaves as $P(x) \sim x$, giving $m(r) \sim r^3$ and producing a regular de Sitter-like core. In the exterior region, corrections to the Schwarzschild geometry decay exponentially.

The logarithmic rate of change of the profile reaches its maximum at $x = 1$. This is the same point selected by the curvature-threshold condition. The coincidence follows from the use of the common variable x and reflects the internal consistency of the parametrization rather than an independent derivation of the exponential profile.

4. Global Curvature Bound

After imposing the relation r_c(M), the Kretschmann scalar factorizes into a dimensional coefficient and a universal dimensionless profile. The mass dependence is transferred entirely to the radial scale.

$$K(r) = (r\_s^2/r\_c^6) \cdot \mathscr{K}(x) \quad (7)$$

Strict monotonicity of the profile $\mathscr{K}(x)$ was established through a method combining three independent components: an exact series estimate near the origin, interval arithmetic on a compact domain, and asymptotic analysis for large x.

1. Exact rational Taylor estimate for $0 < x \leq 0.1$.
2. Outward-rounded interval arithmetic for $0.1 \leq x \leq 6$.
3. Analytic positivity estimate for $x \geq 6$.

GLOBAL CURVATURE BOUND
The curvature profile decreases strictly outward from the centre. Throughout the geometry, $K(r) \leq K(0) = 2\eta K_p$, with equality only at the regular centre.

The result is stronger than the mere finiteness of the central curvature: it excludes any larger maximum away from the centre. For fixed η, the upper bound is independent of mass. Increasing the mass enlarges the transition region but does not increase the maximum curvature.

5. Mass Regimes and Horizon Structure

The number of horizons is controlled by the dimensionless parameter λ = r_s/r_c. Analysis of the global minimum of the horizon function yields an exact classification into three regimes.

Regime	Condition	Structure
Supercritical	M > M_min	outer horizon r_+ and inner horizon r_-
Critical	M = M_min	one degenerate horizon
Subcritical	M < M_min	regular horizonless geometry

For the reference convention η = 1, one obtains $M\_min \approx 1.635621\ M_p$. The critical configuration is not automatically a stable remnant; it is a static degenerate solution whose stability requires a separate perturbative and dynamical analysis.

6. Inner Horizon and Surface Gravity

The transition radius r_c and the inner horizon r_- are determined by different conditions. For most supercritical masses, the inner horizon lies inside the transition region, whereas within a narrow range near the critical configuration it moves to the outer side of the point x = 1.

For every non-extremal two-horizon configuration, the inner-horizon surface gravity is positive. It increases monotonically with mass, vanishes in the extremal limit, and approaches a finite Planck-scale value for large masses.

$$0 < \kappa_-(M) < (\eta/12)^{1/4} \cdot c^2/\ell_p \quad (8)$$

A nonzero κ_- indicates the presence of the kinematic factor associated with the standard blueshift mechanism at a Cauchy horizon. This result alone does not constitute a complete proof of mass inflation, which would require time-dependent perturbations and their backreaction.

For large masses, the inner radius approaches a value of the order of the Planck length. The study shows that this behaviour is not unique to the exponential profile, but follows more generally from regular behaviour of the form $P(x) = ax + O(x^2)$ together with the scaling $r_c^3 \propto r_s \ell_p^2$.

7. Effective Stress-Energy Tensor

The effective stress-energy tensor is defined through the Einstein tensor of the regular geometry. It need not represent a conventional material fluid; it may encode averaged gravitational or semiclassical corrections.

$$\varepsilon(x) = \varepsilon_0 e^{-x}, \quad p_r = -\varepsilon, \quad p_t = -\varepsilon(1 - 3x/2) \quad (9)$$

Condition	Result	Domain / significance
NEC	satisfied	radially saturated and tangentially positive
WEC	satisfied	nonnegative energy density
SEC	violated	central de Sitter region: x < 2/3
DEC	partially violated	tangential tail for x > 4/3; the excess is bounded and vanishes asymptotically

Violation of the strong energy condition in the core is consistent with the negative pressure of a de Sitter-like geometry. Violation of the dominant energy condition was quantitatively bounded: it reaches a finite maximum and vanishes for large x.

8. Physical Significance and Limits of Interpretation

The model shows that the exterior curvature can determine the scale at which the unmodified Schwarzschild continuation reaches a prescribed threshold. The regular metric then provides an effective continuation with globally bounded curvature.

> CAUTIOUS INTERPRETATION
> Finite curvature of the static background does not guarantee dynamical stability of the inner horizon. The location of the horizon relative to the transition region classifies the geometry but does not establish a new instability mechanism.

The study does not derive the exponential profile from a fundamental action, solve dynamical collapse or evaporation, or prove the stability of the critical configuration. It also does not resolve the black-hole information problem.

Its result is more limited but precise: the classical exterior geometry supplies an invariant criterion for determining the transition scale, and the adopted regular continuation permits a global, mass-independent curvature bound to be established.

9. Directions for Further Research

- comparison of the exponential and Hayward profiles under the same threshold prescription;
- a complete analysis of geodesic completeness and maximal extension;
- perturbation equations and the dynamical stability of the inner horizon;
- the semiclassical stress-energy tensor and backreaction near the horizons;
- generalization of the curvature threshold to rotating Kerr-type geometries.

10. Final Conclusion

> SEQUENCE OF RESULTS
> exterior Schwarzschild curvature → transition scale r_c(M) → regular interior with globally bounded curvature

The central consequence is the separation of two scales: the macroscopic transition radius, which grows as $M^{1/3}$, and the maximum core curvature, which remains independent of mass for fixed η. The result provides a controlled bridge between the classical exterior geometry and a phenomenological description of a regular interior, without assigning the present model the status of a complete microscopic theory.

Report basis

Katarzyna Anna Paruzel, Michał Izaak Paruzel, “Planck-Curvature Thresholds and the Limits of Geometric Description in Regular Black Hole Interiors”, Horizon Threshold Project, 2026.